3.7a Implicit Differentiation

differentiate:

$y = 1 - x^2$ explicit $\quad y' = -2x \quad \frac{dy}{dx} = -2x$

$x^2 + y = 1$ implicit $\quad$ ~~$2x + 1 = 0$~~ $\quad 2x + y' = 0 \quad y' = -2x$

$y^2 = x$ implicit $\quad 2yy' = 1$ so $y' = \frac{1}{2y}$

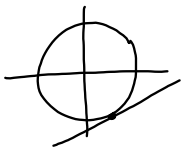
$\frac{d}{dx}(\sin x)^2 = 2\sin x \cdot \cos x$

$\frac{d}{dx}(y)^2 = 2y \cdot y'$

Sep 28-5:35 PM

Find the slope of the circle at the point (3,-4)

$x^2 + y^2 = 25$



$2x + 2yy' = 0$

$2yy' = -2x$

$y' = \frac{-2x}{2y} = \frac{-x}{y}\Big|_{(3,-4)} = \frac{3}{4}$

Sep 28-5:46 PM

Implicit Differentiation Process

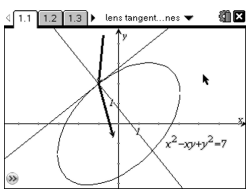
1. Differentiate both sides of the equation with respect to *x*. (term by term)
2. Collect the terms with *dy*/*dx* on one side of the equation
3. Factor out *dy*/*dx*.
4. Solve for *dy*/*dx*.

Sep 28-5:54 PM

lens, tangents and normal lines

find the tangent and normal lines to the ellipse at the point (-1,2)

$x^2 - xy + y^2 = 7$



$2x - (xy' + y \cdot 1) + 2yy' = 0$

$2x - xy' - y + 2yy' = 0$

$-xy' + 2yy' = -2x + y$

$y'(-x + 2y) = -2x + y$

$y' = \frac{-2x+y}{-x+2y}\Big|_{(-1,2)} = \frac{-2(-1)+2}{-(-1)+2\cdot 2} = \frac{4}{5}$

$m_{tan} = \frac{4}{5}$

$m_{\perp} = -\frac{5}{4}$

Sep 28-5:50 PM

higher order derivatives

Find the second derivative of y with respect to x

$2x^3 - 3y^2 = 8$

$6x^2 - 6yy' = 0$

$y' = \frac{-6x^2}{-6y} = \frac{x^2}{y}$

$y'' = \frac{y \cdot 2x - x^2 y'}{y^2}$

$= \frac{y \cdot \left[2x - x^2\left(\frac{x^2}{y}\right)\right]}{y \cdot y^2}$

$= \frac{2xy^2 - x^4}{y^3}$

Sep 28-6:12 PM